[GoGeometry Problem 586]

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Because there are many circles intersect in pairs will use radical axis theory

$AB\cap DC=E$,$BC\cap AD=F$

C: Circumcircle of quadrilateral ABCD center O

C_1: Circumcircle of triangle ABP ,center K_1

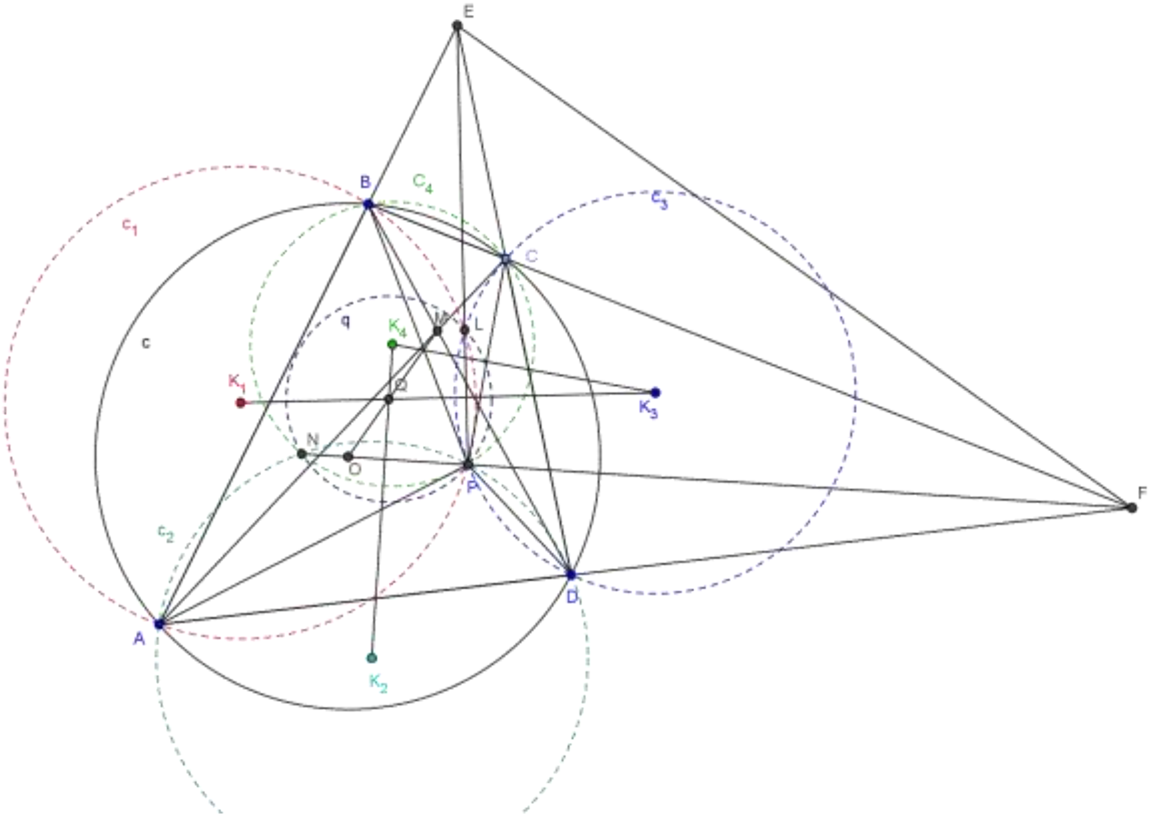
C_2: Circumcircle of triangle APD ,center K_2

C_3: Circumcircle of triangle PDC ,center K_3

C_4: Circumcircle of triangle CBP ,center K_4

q: : Circumcircle of triangle NPL ,center Q

$L= C_1\cap C_3$,$N= C_2\cap C_4$



PN is the radical axis of circle C_2 and C_4. So K_2K_4 is mediator of PN

PL is the radical axis of circle C_1 and C_3 so K_1K_3 is mediator of PL

If $K_1K_3 \cap K_2K_4 = Q$,Q is the center of Circumcircle (q) of triangle LPN.

Therefore, PL is radical axis of the circles q and C_3 and q and C_1

PN is the radical axis of the circles q and C_2 and q and C_4

ABE is the radical axis of the circle c and C_1 , therefore, $\Delta^E_c=\Delta^E_{c1}$

DCE is the radical axis of the circle c and C_2 , therefore $\Delta^E_c=\Delta^E_{c3}$

So , $\Delta^E_{c1} = \Delta^E_{c3}$ .But radical axis o f the circles C_1 and C_3 is PL. So, the point E belongs to PL, which is and radical axis of the circles q and C_1 , q and C_3 .Therefore, $\Delta^E_c= \Delta^E_q$ (1)

Similarly

ADF is the radical axis of the circles c and C_2 , therefore, $\Delta^F_c=\Delta^F_{c2}$

BCF is the radical axis of the circles c and C_4 , therefore $\Delta^F_c=\Delta^F_{c4}$

So , $\Delta^F_{c2} = \Delta^F_{c4}$. But radical axis of the circles C_2 and C_4 is PN. So, the point F belongs to PN ,which is and radical axis of the circles q and C_2 , q and C_4 .Therefore, $\Delta^F_c= \Delta^F_q$ (2)

(1),(2)$\Rightarrow$ The points E and F are on radial axis of the circles c and q. So, EF is radical axis of the circles c,q, therefore, $OQ \perp EF$

The proof will be finished if we prove that $OM\perp$ because in this case, the points O ,Q,M will belong to the same straight

This will prove as an independent problem, in wich I will give very simple solution

Problem

ABCD is cyclic quadrilateral. $AC \cap BD = M$.

c: Circumcircle of quadrilateral ABCD ,center O.

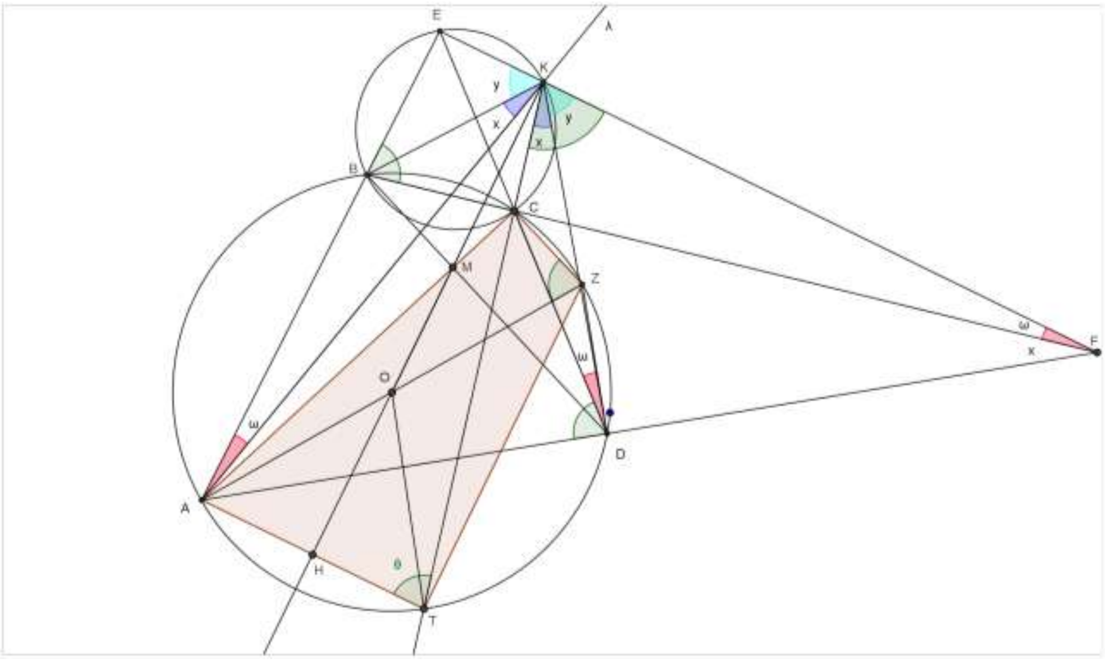
$AB \cap DC = E$ and $BC \cap AD = F$. To prove that $OM \perp EF$

Proof

the circumcircle of the triangle EBC meets at point K the EF

If AO⌒(c)=Z, and KC⌒(c)=T then ∠ATC=∠AZC =∠CDA=∠CBE=∠CKF=θ , because ACZT , ACZD ,ABCD ,BEKC is cyclic quadrilaterals.

Therefore, DCKF is cyclic quadrilateral so, y= ∠DKF= ∠FCD.But ∠FCD = ∠BAD (because, ABCD is cyclic quadrilateral) and ∠DCF= ∠BCE=∠BKE=y.So, BAFK is cyclic quadrilateral



Furthermore ,∠CKD=∠BFA=∠BKA=x. So ∠EKA=∠FKλ=x+y and ∠TKF=θ=x+y.Therefore, the KF is the bisector of ∠TKλ .But ∠KTA=∠TKF=θ=x+y ,so AT//KF and triangle KAT is isosceles⇒ KA=KT . Furthermore OA=OT .Therefore KMOH is the mediator of AH ,so OM⊥EF

O.ε.δ